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A Guide to Contaminant Removal Effectiveness

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Air Infiltration and Ventilation Centre
University of Warwick Science Park
Barclays Venture Centre
Sir William Lyons Road
Coventry CV4 7EZ
Great Britain

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The Air Infiltration and Ventilation Centre
University of Warwick Science Park
Barclays Venture Centre
Sir William Lyons Road
Coventry CV4 7EZ
Great Britain

A Guide to Contaminant Removal Effectiveness

Corine Brouns

(Visiting Specialist from Coventry Polytechnic, UK)

Bob Waters

(Coventry Polytechnic, UK)

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Preface

International Energy Agency

The International Energy Agency (IEA) was established in 1974 within the framework of the Organisation for Economic Co-operation and Development (OECD) to implement an International Energy Programme. A basic aim of the IEA is to foster co-operation among the twenty-one IEA Participating Countries to increase energy security through energy conservation, development of alternative energy sources and energy research development and demonstration (RD&D). This is achieved in part through a programme of collaborative RD&D consisting of forty-two Implementing Agreements, containing a total of over eighty separate energy RD&D projects. This publication forms one element of this programme.

Energy Conservation in Buildings and Community Systems

The IEA sponsors research and development in a number of areas related to energy. In one of these areas, energy conservation in buildings, the IEA is sponsoring various exercises to predict more accurately the energy use of buildings, including comparison of existing computer programs, building monitoring, comparison of calculation methods, as well as air quality and studies of occupancy. Seventeen countries have elected to participate in this area and have designated contracting parties to the Implementing Agreement covering collaborative research in this area. The designation by governments of a number of private organisations, as well as universities and government laboratories, as contracting parties, has provided a broader range of expertise to tackle the projects in the different technology areas than would have been the case if participation was restricted to governments. The importance of associating industry with government sponsored energy research and development is recognized in the IEA, and every effort is made to encourage this trend.

The Executive Committee

Overall control of the programme is maintained by an Executive Committee, which not only monitors existing projects but identifies new areas where collaborative effort may be beneficial. The Executive Committee ensures that all projects fit into a pre-determined strategy, without unnecessary overlap or duplication but with effective liaison and communication. The Executive Committee has initiated the following projects to date (completed projects are identified by *):

I Load Energy Determination of Buildings*
II Ekistics and Advanced Community Energy Systems*
III Energy Conservation in Residential Buildings*
IV Glasgow Commercial Building Monitoring*

V Air Infiltration and Ventilation Centre
VI Energy Systems and Design of Communities*
VII Local Government Energy Planning*
VIII Inhabitant Behaviour with Regard to Ventilation*
IX Minimum Ventilation Rates*
X Building HVAC Systems Simulation*
XI Energy Auditing*
XII Windows and Fenestration*
XIII Energy Management in Hospitals*
XIV Condensation*
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XIX Low Slope Roof Systems
XX Air Flow Patterns within Buildings
XXI Thermal Modelling
XXII Energy Efficient Communities
XXIII Multizone Air Flow Modelling (COMIS)
XXIV Heat Air and Moisture in Envelopes

Annex V Air Infiltration and Ventilation Centre

The IEA Executive Committee (Building and Community Systems) has highlighted areas where the level of knowledge is unsatisfactory and there was unanimous aggreement that infiltration was the area about which least was known. An infiltration group was formed drawing experts from most progressive countries, their long term aim to encourage joint international research and increase the world pool of knowledge on infiltration and ventilation. Much valuable but sporadic and uncoordinated research was already taking place and after some initial groundwork the experts group recommended to their executive the formation of an Air Infiltration and Ventilation Centre. This recommendation was accepted and proposals for its establishment were invited internationally.

The aims of the Centre are the standardisation of techniques, the validation of models, the catalogue and transfer of information, and the encouragement of research. It is intended to be a review body for current world research, to ensure full dissemination of this research and based on a knowledge of work already done to give direction and firm basis for future research in the Participating Countries.

The Participants in this task are: Belgium, Canada, Denmark, Germany, Finland, France, Italy, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States of America.

NOMENCLATURE

Symbols		Units
t	time	s
τ_n	nominal time constant for the ventilation air	s
τ_n^c	nominal time constant for the contaminant	s
$\bar{\tau}_e^c$	local mean age of contaminant at the exhaust duct	s
$\bar{\tau}_p^c$	local mean age of contaminant at point p	s
$\langle \bar{\tau}^c \rangle$	room mean age of contaminant	s
D_p	total dosage index	s
T_{pn}	transfer index	s/m^3 or s/kg
U_p	local purging flow rate	m^3/s
ε^c	contaminant removal effectiveness	
η^c	contaminant removal efficiency	
ε_p^c	local air quality index	
$C_p(t)$	concentration of contaminant at point p at time t	
$C_e(t)$	concentration of contaminant at exhaust at time t	
C_s	concentration of contaminant in supply duct	
$C(0)$	initial concentration of contaminant in room	
$\langle C(t) \rangle$	room mean concentration of contaminant	
C_i	concentration of contaminant in zone i	
V	room volume	m^3
V_c	equivalent volume of contaminant in the room	m^3
V_{ci}	equivalent volume of contaminant in zone i	m^3
Q	airflow rate from supply duct	m^3/s
q_i	injection rate of contaminant in zone i	m^3/s
$\dot{m}_i$	injection rate of contaminant in zone i	kg/s
m_i	quantity of contaminant in zone i	kg
F_{ij}	air flow rate from zone i to zone j	m^3/s

1. INTRODUCTION

The main objective of this report was to provide an introduction to the subject of contaminant removal effectiveness. Existing literature in this subject area is limited, and tends to be very difficult for a newcomer to understand. In recent years, a number of parameters have been defined in order to quantify contaminant removal effectiveness, but not all authors have used the same names or symbols for similar parameters, or derived them in the same way. The usefulness and applicability of the various parameters has not been presented in a comprehensive way in a comparative format and, although the measurement of these parameters has been reported by several authors, there are few published summaries of the most suitable methods. Finally, none of the existing parameters provide a relative measure of contaminant removal effectiveness in the same way as air change efficiency, which provides a comparison with piston flow. Therefore, this report aims to show the origins of the concepts used, provide proofs of the basic formulae and suggests standard symbols and definitions. It also recommends methods of measurement with particular reference to difficulties and possible errors, and investigates the possibility of deriving a contaminant removal effectiveness parameter which will provide a measure of the performance of a ventilation system in removing a contaminant relative to some reference system.

Sandberg and Skåret [3] differentiate between the terms air change efficiency and contaminant removal effectiveness*. Air change efficiency is a measure of how effectively the air present in a room is replaced by fresh air from the ventilation system whereas contaminant removal effectiveness is a measure of how quickly an air-borne contaminant is removed from the room. This report covers only contaminant removal effectiveness and related concepts. It should be noted that the theory and definitions described in this report refer to an air tight room where all the air enters and leaves via single inlet and exhaust ducts. However, these ducts may be taken as the summation of all possible inlet and outlet paths which means that the theory and definitions are applicable to any room, regardless of the method of ventilation.

It should also be noted that in all definitions, net concentrations are used in preference to absolute concentrations. This means that all concentration values are taken as the level above the value in the outside air or supply duct. This simplification causes no loss in generality.
The use of volumetric measures of air and contaminant imply that temperature and pressure are constant throughout the ventilation system. The errors caused by this assumption are sufficiently small to be ignored in the majority of practical cases.

The list of references given at the back of this report is not intended to be exhaustive in the subject area of effectiveness; it is merely a selected list. A complete bibliography can be obtained via AIRBASE.

* Note: Sandberg, Skåret and others often refer to contaminant removal effectiveness as ventilation efficiency.

2. VENTILATION INDICES

Other symbols have been used by different authors. These other symbols are defined in Appendix A. It should be noted that all contaminant concentrations are net values above the concentration level in the air in the supply duct. This is equivalent to setting the concentration in the supply duct, C_s, to zero.

2.1 Nominal Time Constant for the Contaminant (τ_n^c)

This is also called the turnover time of the contaminant, or the transit time for the contaminant flow through the room. The nominal time constant for the contaminant is defined as the ratio between the equivalent volume of contaminant in the room and the contaminant injection rate.

$$\tau_n^c = \frac{V_c}{q} \qquad (2.1)$$

where V_c is the equivalent volume of contaminant in the room (m^3), and q is the contaminant injecton rate (m^3/s).

V_c is defined by the expression:

$$V_c = \langle C(\infty) \rangle \, . \, V \qquad (2.2)$$

Note that the nominal time constant for the contaminant may also be expressed in terms of contaminant mass:

$$\tau_n^c = \frac{m}{\dot{m}} \qquad (2.3)$$

The nominal time constant for the contaminant may also be defined as the average time it takes for the contaminant to flow from its source to the exhaust duct. This will be demonstrated in section 7.3.

2.2 Contaminant Removal Effectiveness (ε^c)

The Contaminant Removal Effectiveness is defined as the ratio between the steady state concentration of contaminant at the exhaust duct and the steady state mean concentration of the room.

$$\varepsilon^c = \frac{C_e(\infty)}{\langle C(\infty) \rangle} \qquad (2.4)$$

From equation 2.2, it can be seen that:

$$<C(\infty)> = \frac{V_c}{V} \tag{2.5}$$

Furthermore,

$$C_e(\infty) = \frac{q}{Q} \tag{2.6}$$

Substituting equations 2.5 and 2.6 into equation 2.4 gives:

$$\varepsilon^c = \left(\frac{q}{Q}\right) \cdot \left(\frac{V}{V_c}\right) = \left(\frac{V}{Q}\right) \cdot \left(\frac{q}{V_c}\right)$$

Substituting equation 2.1 into the above equation gives:

$$\varepsilon^c = \frac{\tau_n}{\tau_n^c} \tag{2.7}$$

where $\tau_n = \dfrac{V}{Q}$ is the nominal time constant for the ventilation air.

Hence, the contaminant removal effectiveness may also be defined as the ratio between the nominal time constant for the ventilation air and the nominal time constant for the contaminant. It will be shown later that the type of ventilation gives rise to values of ε^c as follows:

Complete Mixing: $\varepsilon^c = 1$

Piston Flow : $\varepsilon^c \geq 1$

Short Circuiting : $0 \leq \varepsilon^c \leq 1$

2.3 Contaminant Removal Efficiency (η^c)

The Contaminant Removal Efficiency is derived from the contaminant removal effectiveness.

$$\eta^c = \left(\frac{\varepsilon^c}{1 + \varepsilon^c}\right) \tag{2.8}$$

The values of η^c for different types of ventilation are thus:

Complete Mixing: $\eta^c = 0.5$

Piston Flow : $0.5 \leq \eta^c \leq 1$

Short Circuiting : $0 \leq \eta^c \leq 0.5$

2.4 Local Air Quality Index (ε_p^c)

The Local Air Quality Index is defined as the ratio between the steady state concentration of contaminant at the exhaust duct and the steady state concentration of contaminant at a point p in the room.

$$\varepsilon_p^c = \frac{C_e(\infty)}{C_p(\infty)} \tag{2.9}$$

Note that the contaminant may be injected anywhere within the room.

2.5 Local Purging Flow Rate (U_p)

Consider a small volume, δV, surrounding a point p, as shown in figure 2.1. Assume that the air in this small volume is fully mixed, and that contaminant is injected at a constant rate, q_p. Let U_p be the effective flow rate at which contaminant is removed from the volume. This flow rate, which allows for contaminated air which is recirculated back to δV, is called the local purging flow rate. If $C_p(\infty)$ is the equilibrium concentration within δV due to the injection rate q_p, then a contaminant balance on δV gives:

$$U_p = \frac{q_p}{C_p(\infty)} \tag{2.10}$$

If the total flow rate through the room is Q, and the injection rate q_p leads to a concentration $C_e(\infty)$ in the exhaust duct, then $q_p = Q \, . \, C_e(\infty)$ and U_p may also be written as:

$$U_p = \frac{C_e(\infty)}{C_p(\infty)} \cdot Q \tag{2.11}$$

Note that because, in this case, $C_p(\infty)$ is the concentration due solely to contaminant injected in the volume δV, then $C_p(\infty)$ cannot be less than $C_e(\infty)$. Hence the maximum value of U_p is the fresh air flow rate Q. In the case of piston flow, U_p is equal to the actual flow through δV, and if in addition δV is an elemental plane perpendicular to the flow, then U_p is equal to Q.

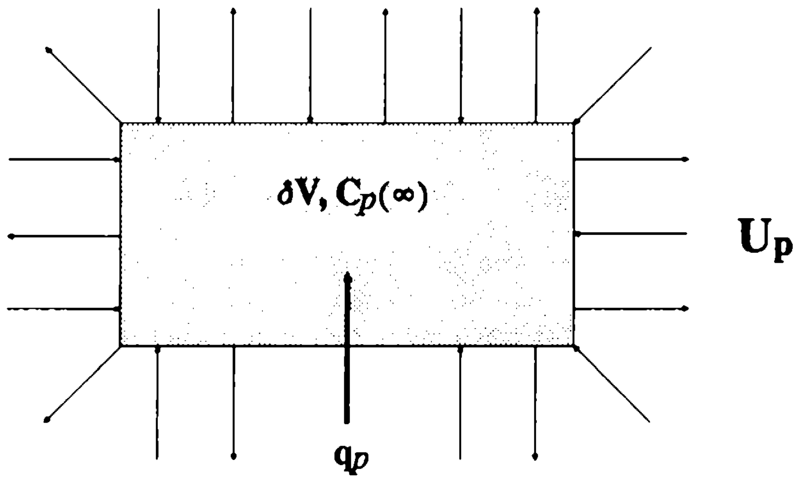


Fig. 2.1 Illustration of the local purging flow rate.

2.6 Total Dosage Index (D_p)

The dosage index is the integral, over a convenient period of time, of the contaminant concentration at a point p. It corresponds to the area under the concentration curve (figure 2.2).

Dosage index, $D_p(\tau) = \int_0^{\tau} C_p(t) \, . \, dt$ (2.12)

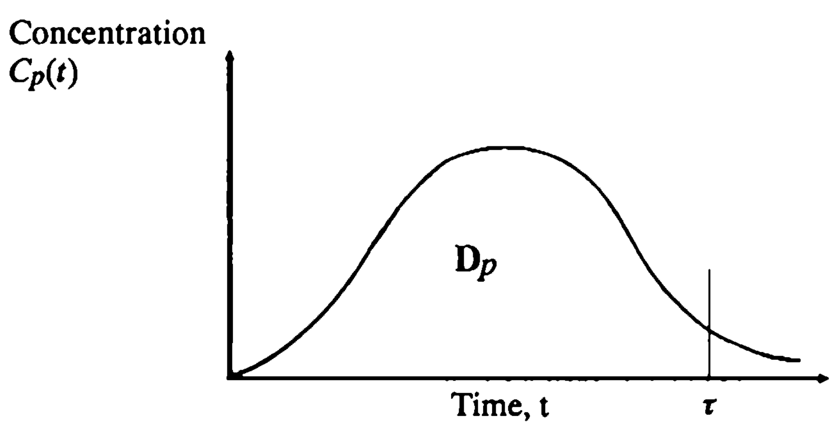


Fig. 2.2 Tracer Concentration Curve

When the integral is taken over all time, that is $\tau = \infty$, the resulting integral is called the Total Dosage Index (or the Total Exposure Index), D_p.

$$D_p = \int_0^{\infty} C_p(t) \, . \, dt \qquad (2.13)$$

Clearly, D_p may be obtained by numerical integration of the measured contaminant concentration curve. Alternatively, consider a small volume, δV, as in figure 2.1. The total equivalent volume of contaminant, V_{cp}, leaving δV, regardless of the manner in which it is injected, must be:

$$V_{cp} = U_p \, . \int_0^\infty C_p(t) \, . \, dt = U_p \, . \, D_p$$

Hence, substituting for U_p from equation 2.10:

$$D_p = V_{cp} \, . \left(\frac{C_p(\infty)}{q_p} \right) \tag{2.14}$$

where $C_p(\infty)$ is the equilibrium concentration in δV due to a continuous contaminant injection rate q_p. Thus, if q_p and its corresponding $C_p(\infty)$ are known, then D_p is the total dosage index due to the release of a quantity V_{cp} of contaminant within δV. The equation may also be expressed in terms of contaminant mass:

$$D_p = m_p \, . \left(\frac{C_p(\infty)}{\dot{m}_p} \right) \tag{2.15}$$

Where the dosage at point p is required due to contaminant release at some other point n, then, provided the fraction of contaminant released at n which reaches p is the same for short term release as for continuous release, equations 2.14 and 2.15 will still hold, that is:

$$D_{pn} = V_{cn} \, . \left(\frac{C_p(\infty)}{q_n} \right) = m_n \, . \left(\frac{C_p(\infty)}{\dot{m}_n} \right) \tag{2.16}$$

The dosage at p due to the release of the same contaminant at several different points may be found by summing the dosages due to each individual point.

2.7 Transfer Index (T_{pn})

The Transfer Index, also called the Index of Exposure to Contamination, is used for describing transport of contaminants between two points in a room. If an equivalent volume of contaminant (V_{cn}) is released suddenly at a point n in the room, and the concentration $C_p(t)$ is measured at a point p in the room, then the Transfer Index, T_{pn}, from point n to point p is defined by:

$$T_{pn} = \frac{\int_0^\infty C_p(t) \, . \, dt}{V_{cn}} \tag{2.17}$$

By using the equations for the total dosage index, we may also express the Transfer Index as:

$$T_{pn} = \frac{D_{pn}}{V_{cn}} = \frac{C_p(\infty)}{q_n} \tag{2.18}$$

where $C_p\,(\infty)$ is the equilibrium concentration at p due to a continuous injection, q_n , at n.

The equations may also be expressed in terms of contaminant mass:

$$T_{pn} = \frac{\int_0^\infty C_p(t)\,.\,dt}{m_n} = \frac{D_{pn}}{m_n} = \frac{C_p(\infty)}{\dot{m}_n} \tag{2.19}$$

2.8 Relationship Between Indices

The indices ε_p^c , U_p , D_p and D_{pn} are inter-related, and may be combined in several ways. However, in doing this, it is necessary to specify the point at which contaminant is injected, especially when the local air quality index is included. Some useful equations are as follows.

(i) Provided ε_p^c is the local air quality index due to contaminant injected at p, then

$$\varepsilon_p^c = \frac{U_p}{Q} = \left(\frac{U_p}{q_p}\right).\,C_e\,(\infty)$$

and

$$\varepsilon_p^c = \frac{V_{cp}}{Q\,.\,D_p} = \left(\frac{V_{cp}}{q_p\,.\,D_p}\right).\,C_e\,(\infty)$$

(ii) Provided ε_p^c is the local air quality index due to contaminant injected at n, then

$$\varepsilon_p^c = \frac{1}{Q\,.\,T_{pn}}$$

3. THE EVALUATION OF VENTILATION INDICES FOR TWO IDEALISED CASES.

The Contaminant Removal Effectiveness and its associated indices can be determined for the idealised cases of complete mixing (paragraph 3.1) and piston flow (paragraph 3.2). These idealised cases form a basis for judging the performance of real ventilation systems.

3.1 Complete Mixing

Under conditions of complete mixing, incoming air continuously and uniformly mixes with the room air, causing the contaminant concentration to be the same everywhere, regardless of the position at which the contaminant is injected.

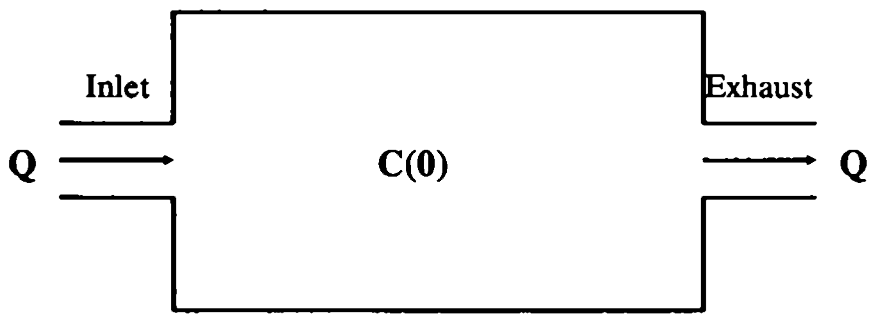


Fig. 3.1 Complete Mixing

As the concentration of the contaminant is the same everywhere, the concentration at the exhaust duct is equal to the concentration averaged over the whole room.

Hence:

$$C_p(\infty) = C_e(\infty) = \langle C(\infty) \rangle$$

and

$$q_p \,(or\ q_n) = Q \,.\, C_e(\infty)$$

Substitution in the defining equations gives immediately:

$$\varepsilon^c = 1$$

$$\eta^c = 0.5$$

$$\varepsilon_p^c = 1$$

$$U_p = Q$$

$$D_p = \frac{V_{cp}}{Q}$$

$$D_{pn} = \frac{V_{cn}}{Q}$$

$$T_{pn} = \frac{1}{Q}$$

3.2 Piston Flow

This is also known as 'plug flow' or 'displacement flow' and is often regarded as the most efficient form of ventilation. The ventilation airflow acts as a piston, which pushes the 'old' air in the room in front of it without actually mixing. In this case, the contaminant concentration varies according to the manner in which the contaminant is injected. Four patterns of contaminant injection are considered, corresponding to the most common cases met in practice. These are:

(i) uniform injection throughout the whole space,

(ii) localised injection across a plane,

(iii) uniform injection in a region close to the inlet duct, and

(iv) uniform injection in a region close to the exhaust duct.

3.21 Uniform Injection Throughout the Whole Space

Contaminant is injected at a constant rate at all points throughout the space to produce a uniform injection in the room.This is illustrated in figure 3.2.

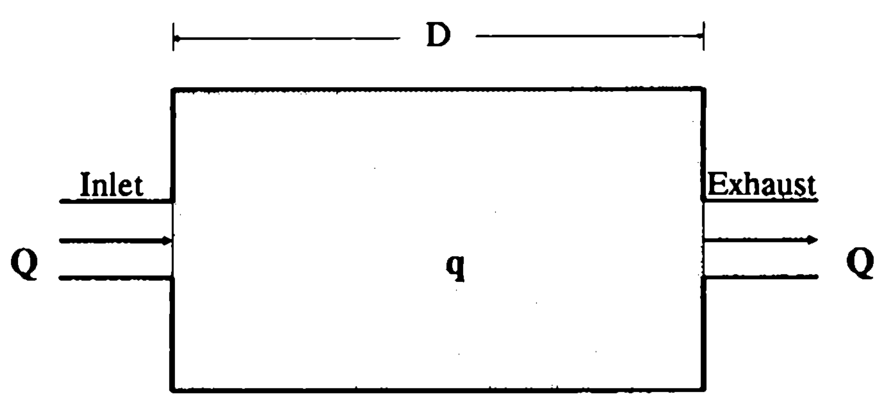


Fig. 3.2 Piston Flow - Uniform Injection Throughout the Whole Space

The concentration of the contaminant in the room varies according to the distance l of point p from the inlet duct, as shown in figure 3.3.

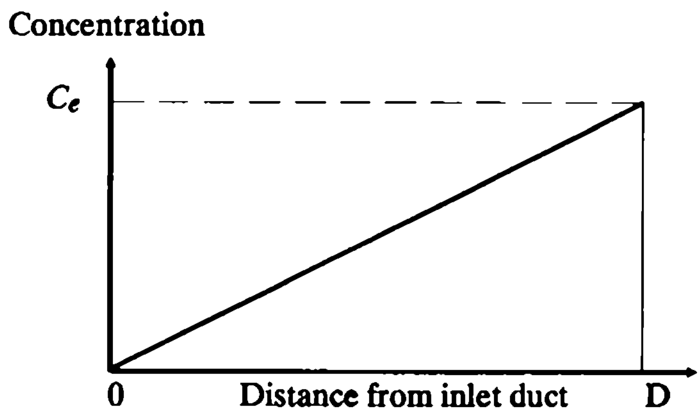


Fig. 3.3 Concentration of Contaminant along the room.

It can be seen from figure 3.3 that the contaminant concentrations are:

$$C_p(\infty) = \frac{l \,.\, C_e(\infty)}{D}$$

$$<C(\infty)> = \frac{C_e(\infty)}{2}$$

Substitution in the defining equations gives:

$$\varepsilon^c = 2$$

$$\eta^c = 0.67$$

$$\varepsilon_p^c = \frac{D}{l}$$

The values of other indices depend on the release of contaminant at a specific point, and not on the overall pattern of contaminant release. Thus, for all cases of piston flow:

$$U_p = Q$$

$$D_p = \frac{V_{cp}}{Q}$$

$$D_{pn} = \frac{V_{cn}}{Q}$$ Provided point n is nearer to the inlet than point p.

$$T_{pn} = \frac{1}{Q}$$ Provided point n is nearer to the inlet than point p.

$$D_{pn} = T_{pn} = 0$$ Provided point p is nearer to the inlet than point n.

3.22 Localised Injection in a Plane across the Room

Contaminant is injected uniformly at a constant rate across a plane of thickness δl at a distance l from the inlet duct, as shown in figure 3.4.

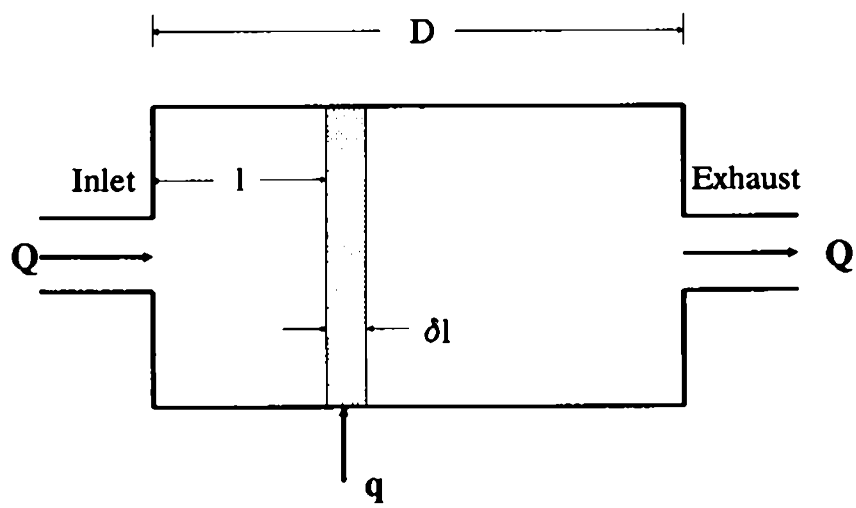


Fig. 3.4 Piston Flow - Localised Injection in a Plane across the Room.

The concentration of the contaminant in the room is a function of the distance of the source of injection from the inlet duct. This is shown in figure 3.5.

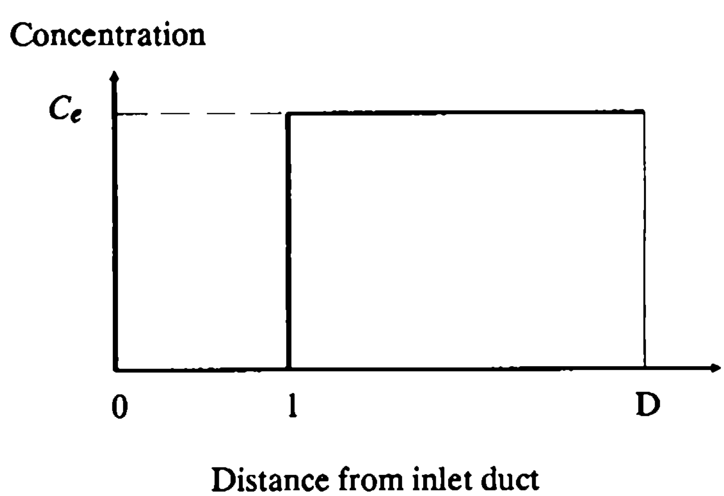


Fig. 3.5 Concentration of contaminant along the room

The contaminant concentrations are:

$C_p(\infty) = 0$ for points between 0 and l

$C_p(\infty) = C_e(\infty)$ for points between l and D

$$<C(\infty)> = \frac{(D - l).C_e(\infty)}{D}$$

Substitution in the defining equations gives:

$$\varepsilon^c = \frac{D}{D - l}$$

$$\eta^c = \frac{D}{2D - l}$$

$\varepsilon_p^c = \infty$ for points between 0 and l

$\varepsilon_p^c = 1$ for points between l and D

The equations for U_p, D_p, D_{pn} and T_{pn} are the same as in paragraph 3.21 above.

3.23 Uniform Injection between the Inlet duct and a Plane across the Room

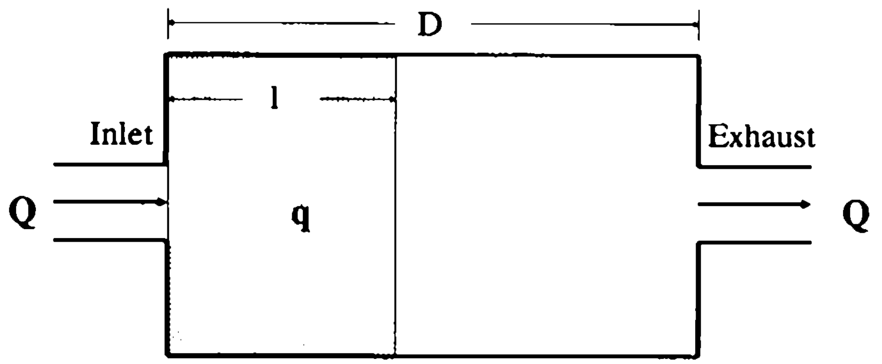


Fig. 3.6 Piston Flow - Uniform Injection between the Inlet duct and a Plane across the Room.

Contaminant is released uniformly throughout the part of the room between the inlet duct and a plane across the room, as shown in figure 3.6. The concentration of the contaminant along the length of the room is illustrated in figure 3.7.

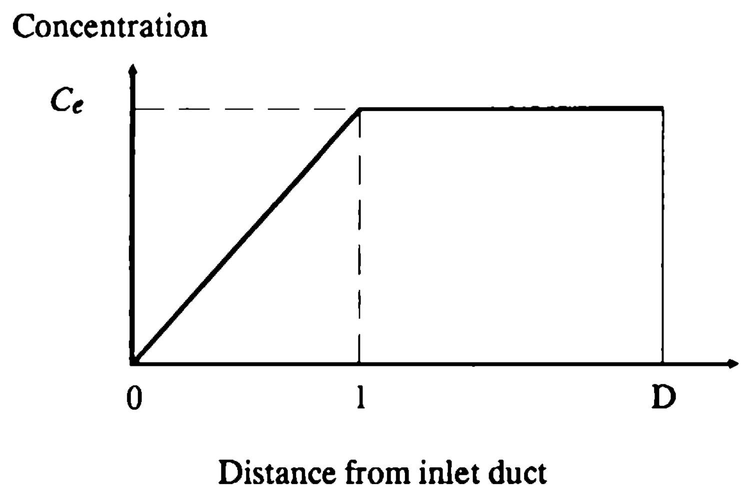


Fig. 3.7 Concentration of contaminant along the room

Let the point p be a distance x from the inlet. The contaminant concentrations are then:

$$C_p(\infty) = \frac{x \,.\, C_e(\infty)}{l} \qquad \text{for } x < 1$$

$$C_p(\infty) = C_e(\infty) \qquad \text{for } x \geq 1$$

$$<C(\infty)> = \left[\frac{l}{2D} + \frac{D-l}{D}\right] . \, C_e(\infty)$$

Substitution in the defining equations gives:

$$\varepsilon^c = \frac{2D}{2D - l}$$

$$\eta^c = \frac{2D}{4D - l}$$

$$\varepsilon_p^c = \frac{l}{x} \qquad \text{for } x < 1$$

$$\varepsilon_p^c = 1 \qquad \text{for } x \geq 1$$

The equations for U_p, D_p, D_{pn} and T_{pn} are the same as in paragraph 3.21 above.

3.24 Uniform Injection between a Plane across the Room and the Exhaust Duct

Contaminant is released uniformly throughout the part of the room between a plane across the room and the exhaust duct, as shown in figure 3.8.

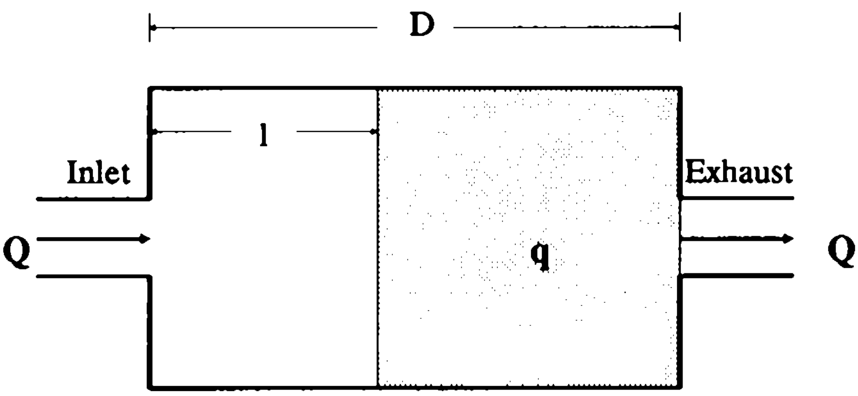


Fig. 3.8 Piston Flow - Uniform Injection between a Plane across the Room and the Exhaust duct.

The variation in the concentration of contaminant along the length of the room is shown in figure 3.9.

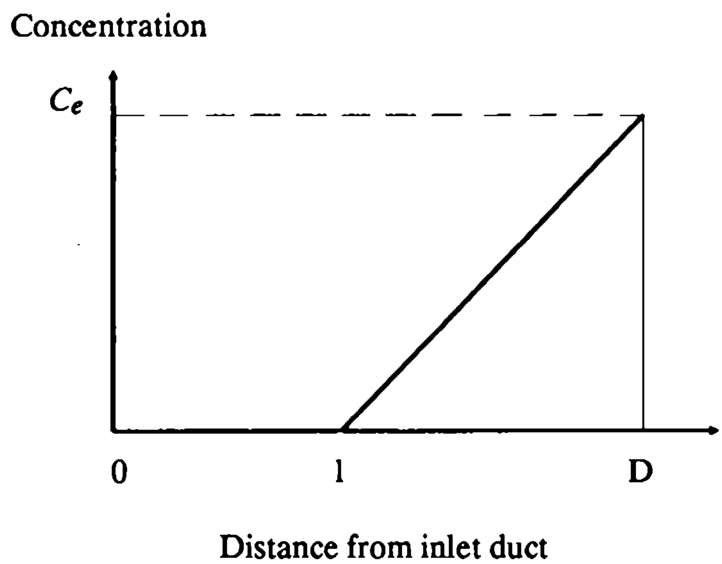


Fig. 3.9 Concentration of contaminant along the room.

Let the point p be a distance x from the inlet. The contaminant concentrations are then:

$$C_p(\infty) = 0 \qquad \text{for } x < l$$

$$C_p(\infty) = \frac{(x - l) \, . \, C_e(\infty)}{(D - l)} \qquad \text{for } x \geq 1$$

$$<C(\infty)> = \frac{(D - l) \, . \, C_e(\infty)}{2D}$$

Substitution in the defining equations gives:

$$\varepsilon^c = \frac{2D}{D - l}$$

$$\eta^c = \frac{2D}{3D - l}$$

$$\varepsilon_p^c = \infty \qquad \text{for } x < 1$$

$$\varepsilon_p^c = \left(\frac{D - l}{x - l}\right) \qquad \text{for } x \geq 1$$

The equations for U_p, D_p, D_{pn} and T_{pn} are the same as in paragraph 3.21 above.

3.3 Discussion

The results for contaminant removal effectiveness and contaminant removal efficiency are shown in figures 3.10 and 3.11.

The results show that for piston flow the contaminant removal effectiveness is always greater or equal to 1, i.e. $\varepsilon^c \geq 1$, and that ε^c increases as the point at which the contaminant is injected moves towards the exhaust duct. This shows that piston flow is always better than fully mixed flow. However, the contaminant concentrations experienced by an occupant in piston flow may be the same as in fully mixed flow. This will occur when the occupant is positioned between the contaminant source and the exhaust duct, and the local air quality index, $\varepsilon_p^c = 1$. It follows, therefore, that the advantages of piston flow are only fully realised when all or part of the airborne contaminants are generated between the occupants and the exhaust duct. It is also clear that the contaminant removal effectiveness of a ventilated space does not normally have a single value, and in most cases varies according to the position of the contaminant source. The interpretation of ventilation indices is considered in detail in section 9.

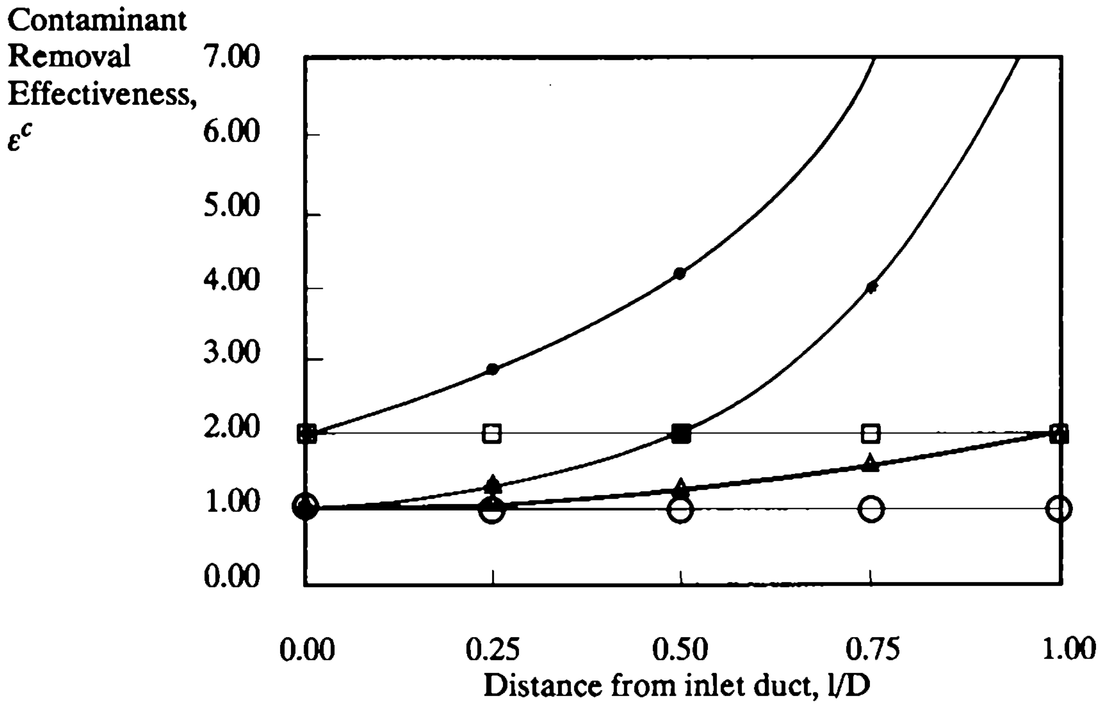


Figure 3.10 - Contaminant Removal Effectiveness vs distance of contaminant source from inlet.

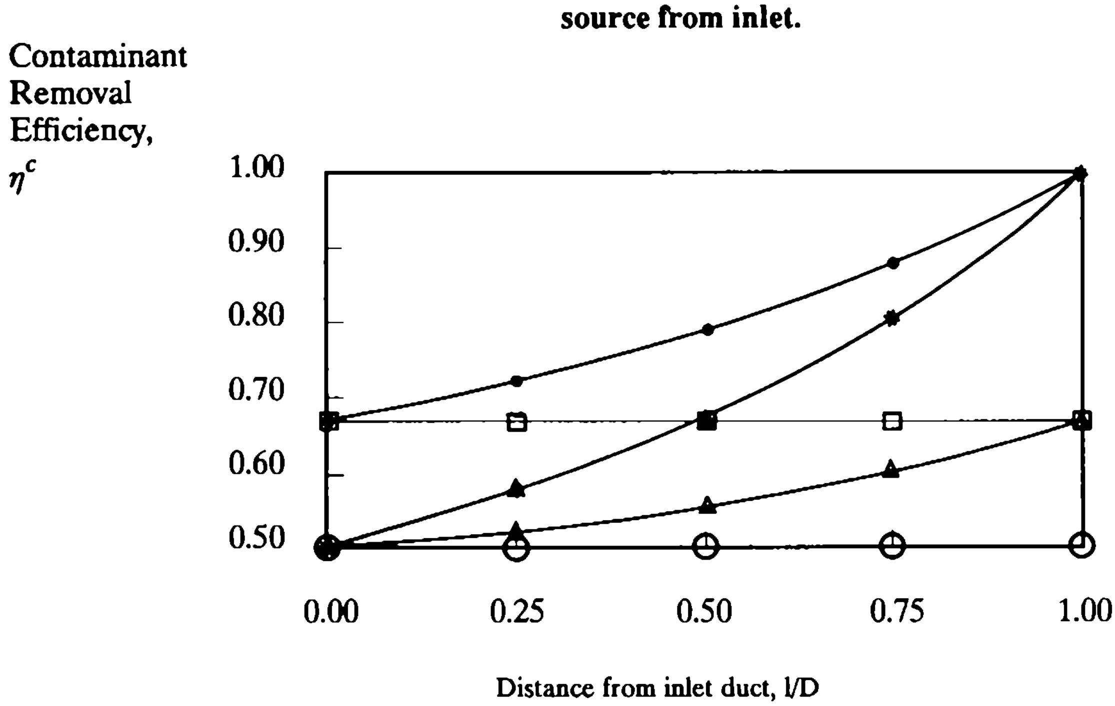


Figure 3.11 - Contaminant Removal Efficiency vs distance of contaminant source from inlet.

Key: ----O---- case 3.1; -----□----- case 3.21; ------✱------ case 3.22; ------▲----- case 3.23; -----●------ case 3.24

4. EVALUATION OF VENTILATION INDICES FOR A TWO ZONE MIXING MODEL

In section 3, the room was treated as a single zone. In this section, the room is considered as two zones, with an imaginary partition between them, as shown in figure 4.1. The air in each zone is assumed to be fully mixed, and recirculation occurs between the zones. One of the zones may be considered as the occupied space (zone 2), and the other (zone 1) as the unoccupied space.

4.1 Theory

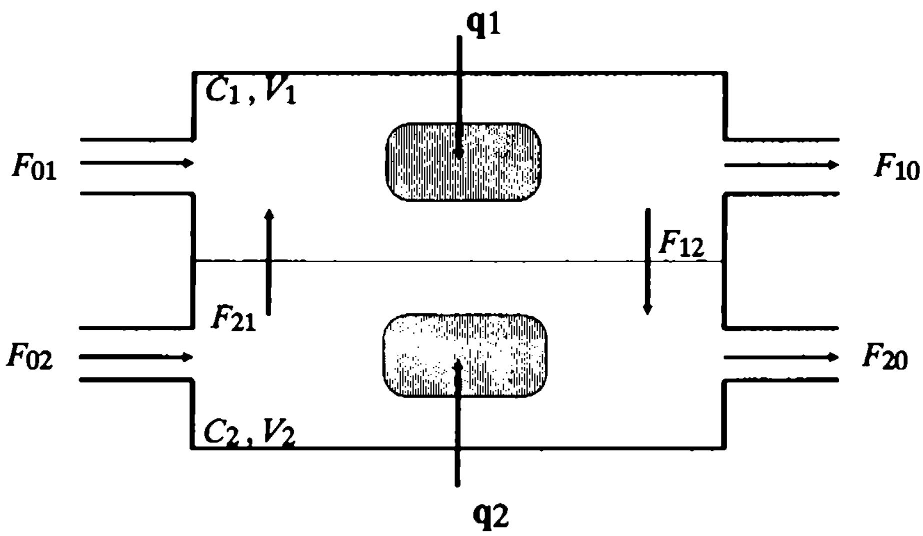


Fig. 4.1 Schematic diagram of a two zone model.

The contaminant balance equation for zone 1 is:

$$F_{01} . C_s + F_{21} . C_2 + q_1 = C_1 . S_1 \qquad (4.1)$$

where $S_1 = F_{10} + F_{12}$

The contaminant balance equation for zone 2 is:

$$F_{02} . C_s + F_{12} . C_1 + q_2 = C_2 . S_2 \qquad (4.2)$$

where $S_2 = F_{20} + F_{21}$

Taking the concentration at the inlet duct(s), C_s as zero, leads to:

$$C_1 = \frac{q_1 . S_2 + q_2 . F_{21}}{S_1 . S_2 - F_{12} . F_{21}} \qquad (4.3)$$

and $$C_2 = \frac{q_1 . F_{12} + q_2 . S_1}{S_1 . S_2 - F_{12} . F_{21}} \tag{4.4}$$

The mean room contaminant concentration, $< C(\infty) >$, is:

$$<C(\infty)> = \frac{C_1 . V_1 + C_2 . V_2}{V_1 + V_2}$$

Substituting for C_1 and C_2 gives:

$$<C(\infty)> = \frac{q_1 . [S_2 . V_1 + F_{12} . V_2] + q_2 . [F_{21} . V_1 + S_1 . V_2]}{(V_1 + V_2) . [S_1 . S_2 - F_{12} . F_{21}]} \tag{4.5}$$

The mean contaminant concentration in the exhaust duct, $C_e(\infty)$, is:

$$C_e(\infty) = \frac{F_{10} . C_1 + F_{20} . C_2}{F_{10} + F_{20}}$$

Substituting for C_1 and C_2 gives

$$C_e(\infty) = \frac{q_1 . [F_{10} . S_2 + F_{20} . F_{12}] + q_2 [F_{10} . F_{21} + F_{20} . S_1]}{(F_{10} + F_{20}) . [S_1 . S_2 - F_{12} . F_{21}]} \tag{4.6}$$

The contaminant removal effectiveness is thus:

$$\varepsilon^c = \frac{C_e(\infty)}{< C(\infty) >}$$

$$= \frac{(V_1+V_2) [q_1 (S_2 F_{10} + F_{20} F_{12}) + q_2 (F_{10} F_{21} + S_1 F_{20})]}{(F_{10}+F_{20}) [q_1 (S_2 V_1 + F_{12} V_2) + q_2 (F_{21} V_1 + S_1 V_2)]} \tag{4.7}$$

4.2 Application of the Theory

The effect of varying the position of the contaminant source, and the effect of varying the position of the outlet duct with respect to the inlet duct, may be evaluated by choosing suitable values for the flow rates. The following examples, in which it is assumed that $V_1 = V_2$, illustrate the analysis of short-circuiting flow and piston flow.

4.21 Short-Circuiting Flow

In this case, fresh air is supplied to and removed from the unoccupied zone (zone 1), and is able to enter the occupied zone (zone 2) only by recirculation. Thus some of the fresh air is able to short circuit the occupied zone.

Two cases of short-circuiting flow can be studied. For both cases, it is assumed that:

$$F_{02} = F_{20} = 0$$
$$F_{01} = F_{10} = Q$$
$$F_{12} = F_{21} = \beta\, F_{01} = \beta Q$$

where β, which is often called the recirculation factor, is a measure of the degree of internal mixing between zone 1 and zone 2.

(a) Contaminant Injection in zone 1:

Substituting the above parameters into equations 4.3 and 4.4, and setting $q_2 = 0$ gives:

$$C_1 = \frac{q_1}{Q} \quad \text{and} \quad C_2 = \frac{q_1}{Q}$$

Hence: $$C_p = C_e(\infty) = C_1 = C_2 = \langle C(\infty) \rangle$$

Substitution in the defining equations gives:

$$\varepsilon^c = 1$$

$$\eta^c = 0.5$$

$$\varepsilon_p^c = 1$$

$$U_p = Q \qquad \text{(in zone 1)}$$

$$D_p = \frac{V_{cp}}{Q}$$

$$D_{pn} = \frac{V_{cn}}{Q}$$

$$T_{pn} = \frac{1}{Q}$$

(b) Contaminant Injection in Zone 2:

Substituting the above parameters into equations 4.3 and 4.4 and setting $q_1 = 0$ gives:

$$C_1 = \frac{q_2}{Q} \quad \text{and} \quad C_2 = \frac{q_2}{Q} \cdot \left(\frac{1+\beta}{\beta}\right)$$

Hence: $$C_e(\infty) = C_1 = \frac{q_2}{Q}$$

$$< C(\infty) > = \frac{q_2}{Q} \cdot \left(\frac{1+2\beta}{\beta}\right)$$

Substitution in the defining equations gives:

$$\varepsilon^c = \frac{2\beta}{1+2\beta}$$

$$\eta^c = \frac{2\beta}{1+4\beta}$$

In zone 1, $C_p = C_1$, and so:

$$\varepsilon_p^c = 1$$

$$D_{pn} = \frac{V_{cn}}{Q}$$

$$T_{pn} = \frac{1}{Q}$$

In zone 2, $C_p = C_2$, and so:

$$\varepsilon_p^c = \frac{\beta}{1+\beta}$$

$$U_p = \left(\frac{\beta}{1+\beta}\right) \cdot Q$$

$$D_p = \frac{V_{cp}}{Q} \cdot \left(\frac{1+\beta}{\beta}\right)$$

$$T_{pn} = \frac{1}{Q} \cdot \left(\frac{1+\beta}{\beta}\right)$$

4.22 Piston Flow

In this case, fresh air enters the occupied zone and is removed from the unoccupied zone. This provides a flow regime with similar, though not identical characteristics to true piston flow.

Two cases of piston flow can be studied. For both cases, it is assumed that:

$$F_{02} = F_{10} = 0$$
$$F_{01} = F_{20} = Q$$
$$F_{12} = (1+\beta) F_{01} = (1+\beta) Q$$
$$F_{21} = \beta F_{01} = \beta Q$$

(a) Contaminant Injection in Zone 1:

Substituting the above parameters into equations 4.3 and 4.4 and setting $q_2 = 0$ gives:

$$C_1 = \frac{q_1}{Q} \quad \text{and} \quad C_2 = \frac{q_1}{Q}$$

Hence: $C_p = C_e(\infty) = C_1 = C_2 = \langle C(\infty) \rangle$

Substitution in the defining equations gives identical results to short-circuiting flow (4.21 (a)) above.

(b) Contaminant Injection in Zone 2:

Substituting the above parameters into equations 4.3 and 4.4 and setting $q_2 = 0$ gives:

$$C_1 = \frac{q_2}{Q} \cdot \left(\frac{\beta}{1+\beta}\right) \quad \text{and} \quad C_2 = \frac{q_2}{Q}$$

Hence: $$C_e(\infty) = C_2 = \frac{q_2}{Q}$$

$$< C(\infty) > = \frac{q_2}{Q} \cdot \left(\frac{1+2\beta}{1+\beta}\right)$$

Substitution in the defining equations gives:

$$\varepsilon^c = 2 \, . \left(\frac{1+\beta}{1+2\beta}\right)$$

$$\eta^c = 2 \, . \left(\frac{1+\beta}{3+4\beta}\right)$$

In zone 1, $C_p = C_1$, and so:

$$\varepsilon_p^c = \frac{1+\beta}{\beta}$$

$$D_{pn} = \frac{V_{cn}}{Q} \cdot \left(\frac{\beta}{1+\beta}\right)$$

$$T_{pn} = \frac{1}{Q} \cdot \left(\frac{\beta}{1+\beta}\right)$$

In zone 2, $C_p = C_2$, and so:

$$\varepsilon_p^c = 1$$

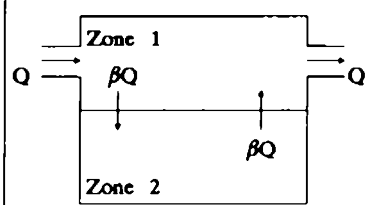
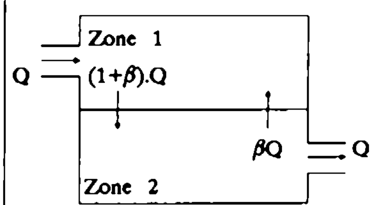
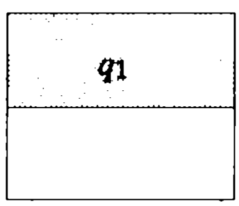
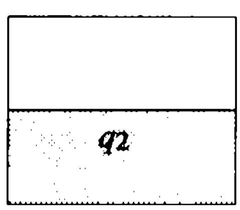
$$U_p = Q$$

$$D_p = \frac{V_{cp}}{Q}$$

$$T_{pn} = \frac{1}{Q}$$

Table 4.1 compares the results for contaminant removal effectiveness and contaminant removal efficiency.

Table 4.1 - Evaluation of ventilation indices for a two zone model.

EVALUATION OF VENTILATION INDICES FOR A TWO ZONE MIXING MODEL.	Short-Circuiting Flow 	Piston Flow 
a. Contaminant injected in zone 1. 	$C_1 = \frac{q_1}{Q} = C_e(\infty)$ $< C(\infty) > = \frac{q_1}{Q}$ $\varepsilon^c = 1$ $\eta^c = 0.5$	$C_2 = \frac{q_1}{Q} = C_e(\infty)$ $< C(\infty) > = \frac{q_1}{Q}$ $\varepsilon^c = 1$ $\eta^c = 0.5$
b. Contaminant injected in zone 2. 	$C_1 = \frac{q_2}{Q} = C_e(\infty)$ $< C(\infty) > = \frac{q_2}{2Q} \cdot \left(\frac{1+\beta}{\beta}\right)$ $\varepsilon^c = \frac{2\beta}{1+2\beta}$ $\eta^c = \frac{2\beta}{1+4\beta}$	$C_2 = \frac{q_2}{Q} = C_e(\infty)$ $< C(\infty) > = \frac{q_2}{2Q} \cdot \left(\frac{1+2\beta}{1+\beta}\right)$ $\varepsilon^c = 2 . \left(\frac{1+\beta}{1+2\beta}\right)$ $\eta^c = 2 . \left(\frac{1+\beta}{3+4\beta}\right)$

4.3 Discussion

Thecontaminant removal effectiveness and the contaminant removal efficiency are plotted as a function of the re-circulation factor in figures 4.2 and 4.3.

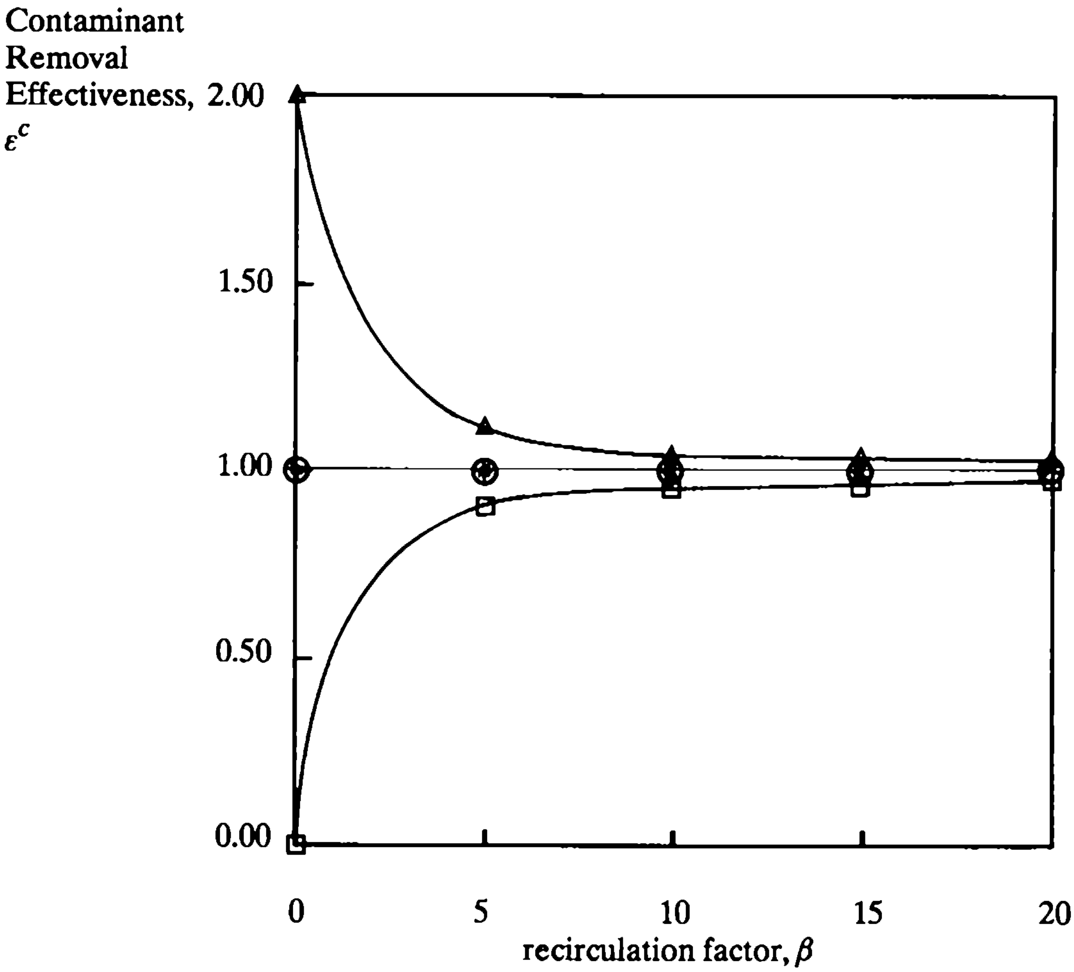


Figure 4.2 - Contaminant removal effectiveness vs recirculation factor.

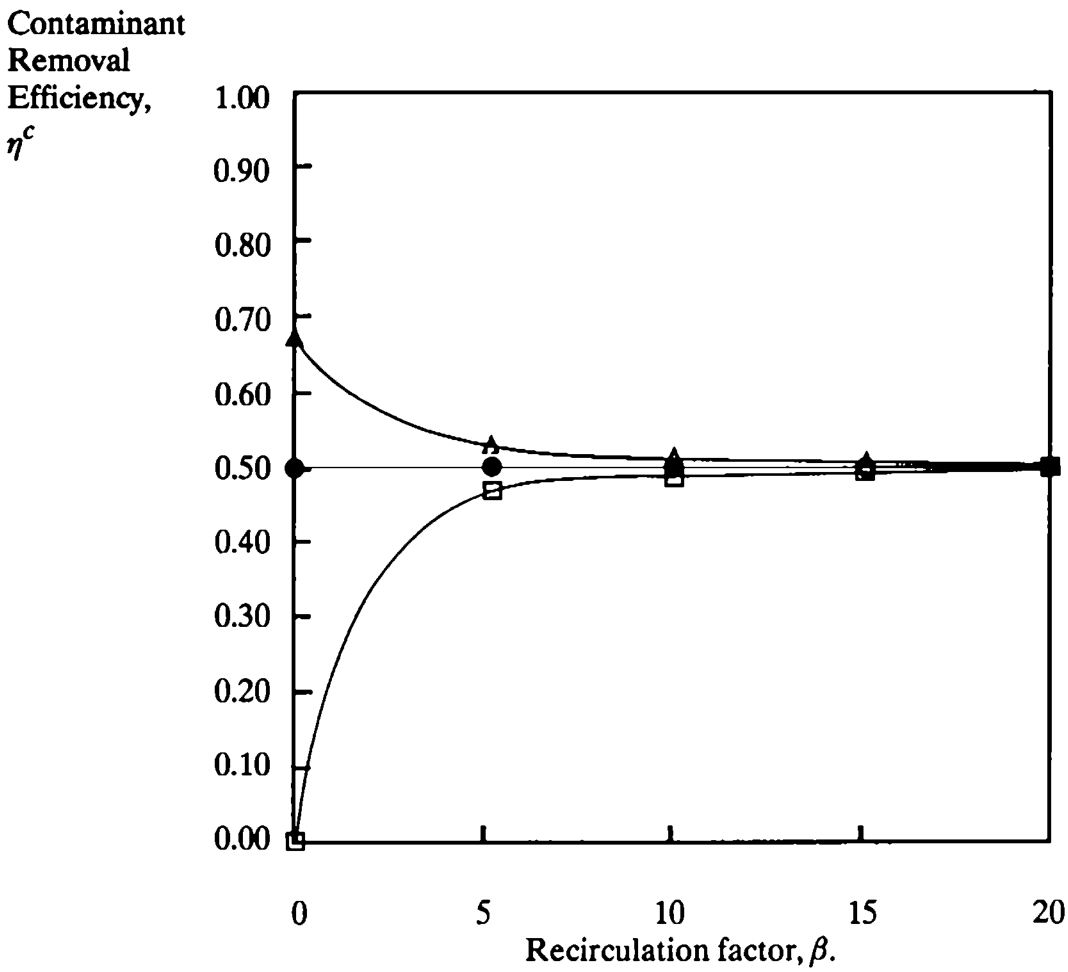


Figure 4.3 - Contaminant removal efficiency vs recirculation factor.

Key: -----●------ case 4.21a; ------□----- case 4.21b; -----♦------ case 4.22a; ------▲------- case 4.22b

It was shown in section 3.1 that, for a fully mixed single zone, $\varepsilon^c = 1$ and $\eta^c = 0.5$. The same result is obtained for short-circuiting flow when the contaminant is injected in the unoccupied zone, showing that these two cases have the same effect. If the contaminant is injected into the occupied zone, then ε^c is always less than 1, showing that the ventilation of the room is worse than the fully mixed case. As the recirculation factor increases, this short-circuited case approaches the fully mixed case. The results for piston flow show that $\varepsilon^c = 1$ when the contaminant is injected into the occupied zone, and $\varepsilon^c > 1$ when injection is in the unoccupied zone. However, the two zone model does not provide as realistic an analysis of piston flow as the treatment gives in section 3.2. The results also show that when the recirculation factor, β, exceeds a value of 4.5, the value of ε^c is always within 10% of its value for fully mixed flow.

5. EVALUATION OF VENTILATION INDICES FOR MULTIZONE MIXING MODELS

The two zone representation of a room used in section 4 can be extended to any number of zones. Increasing the number of zones improves the modelling of the variations in contaminant concentration and air flows in the room. The level of improvement brought about by increasing the number of zones depends on the details of the flow regime which is being analysed. As an example, the contaminant removal effectiveness and contaminant removal efficiency are evaluated for the four zone model shown in figure 5.1.

5.1 Theory

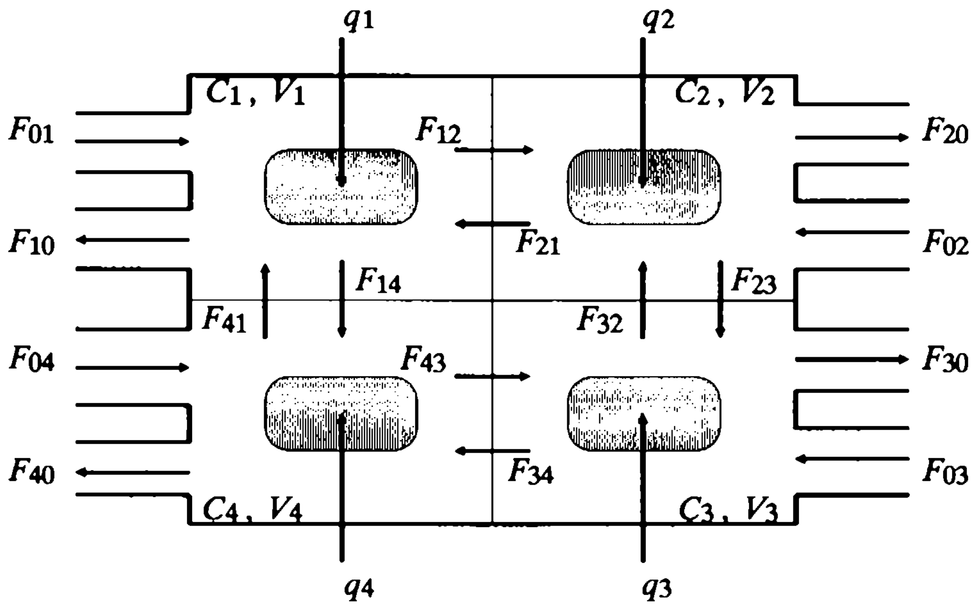


Fig. 5.1 Schematic Diagram of a 4 Zone Model

The contaminant balance equation for zone 1 may be written:

$$F_{01} . C_s + q_1 + F_{21} . C_2 + F_{31} . C_3 + F_{41} . C_4 = C_1 . S_1$$

where $S_1 = F_{10} + F_{12} + F_{13} + F_{14}$

This may be re-arranged in the form:

$$F_{01} . C_s - S_1 . C_1 + F_{21} . C_2 + F_{31} . C_3 + F_{41} . C_4 = - q_1$$

Similar equations may be written for the other zones. The complete set of equations for the room is thus:

$$F_{01} . C_s - S_1 . C_1 + F_{21} . C_2 + F_{31} . C_3 + F_{41} . C_4 = - q_1$$

$$F_{02} . C_s + F_{12} . C_2 - S_2 . C_2 + F_{32} . C_3 + F_{42} . C_4 = - q_2$$

$$F_{03} . C_s + F_{13} . C_1 + F_{23} . C_2 - S_3 . C_3 + F_{43} . C_4 = - q_3$$

$$F_{04} . C_s + F_{14} . C_1 + F_{24} . C_2 + F_{34} . C_3 - S_4 . C_4 = - q_4$$

This is more conveniently expressed in the matrix form

$$\underline{F} . \underline{C} = \underline{q}$$

where, with $C_s = 0$, the matrices are:

$$\underline{F} = \begin{pmatrix} -S_1 & F_{21} & F_{31} & F_{41} \\ F_{12} & -S_2 & F_{32} & F_{42} \\ F_{13} & F_{23} & -S_3 & F_{43} \\ F_{14} & F_{24} & F_{34} & -S_4 \end{pmatrix}$$

$$\underline{C} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \quad \text{and} \quad \underline{q} = \begin{pmatrix} -q_1 \\ -q_2 \\ -q_3 \\ -q_4 \end{pmatrix}$$

The equations may be solved for the contaminant concentrations in each zone by writing:

$$\underline{C} = \underline{F}^{-1} . \underline{q}$$

The room average concentration is:

$$<C(\infty)> = \frac{C_1 . V_1 + C_2 . V_2 + C_3 . V_3 + C_4 . V_4}{V}$$

where $V = V_1 + V_2 + V_3 + V_4$

The average concentration at the exhaust duct is:

$$C_e(\infty) = \frac{F_{10} . C_1 + F_{20} . C_2 + F_{30} . C_3 + F_{40} . C_4}{S_0}$$

where $S_0 = F_{10} + F_{20} + F_{30} + F_{40}$

Substituting in equation 2.4 gives:

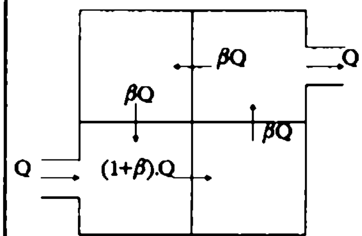
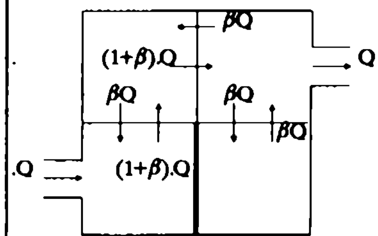
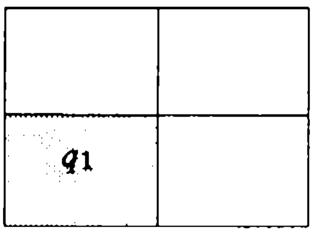
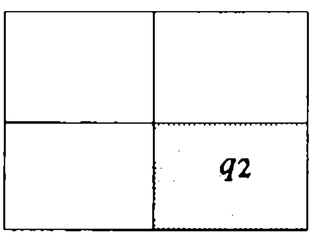
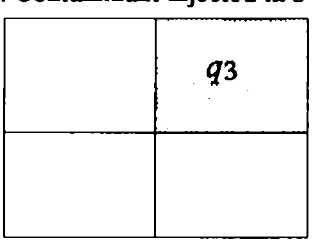
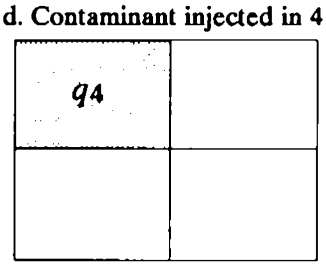
$$\varepsilon^c = \left(\frac{F_{10}\, C_1 + F_{20}\, C_2 + F_{30}\, C_3 + F_{40}\, C_4}{V_1\, C_1 + V_2\, C_2 + V_3\, C_3 + V_4\, C_4} \right) \cdot \frac{V}{S_0}$$

5.2 Examples and Discussion

Two examples have been chosen to illustrate the four zone model. The first example shows strong similarities with the two zone model, and shows very little improvement over two zones. The second example shows some significant differences and is thus a case where the four zone model is necessary. In both cases, it is assumed that the volumes are equal and the contaminant concentration in the inlet duct is zero. The two examples are given in table 5.1, and the results for these examples are plotted in figures 5.2 and 5.3.

When contaminant is injected close to the inlet duct, that is in zone 1, the contaminant removal effectiveness,$\varepsilon^c = 1$, and is independent of the recirculation factor. This is true for all versions of the two and four zone models, and is obviously a result which will apply to any system and number of zones. Similarly, when contaminant is injected close to the outlet duct, that is in zone 3 in the four zone model, the contaminant removal effectiveness, $\varepsilon^c \geq 1$. This again is a result of general validity. However, when an impermeable partition is placed between zones 1 and 2 in the four zone model, the contaminant removal effectiveness shows a substantial change for the cases when the contaminant is injected in either zone 2 or zone 4. For injection in zone 4, ε^c is always less than 1 for the recirculation case, and ε^c is always greater than 1 for the partitioned case. A more complex result occurs with injection in zone 2, where for the recirculation case, ε^c is always greater than 1, but for the partitioned case, ε^c is less than 1 at low values of β, but is greater than 1 when $\beta > \frac{\sqrt{2}}{2}$. It may also be noted that the four zone model is within 10% of the fully mixed case when β is greater than approximately 5.

Table 5.1 - Evaluation of ventilation indices for two four zone models.

EVALUATION OF VENTILATION INDICES FOR TWO FOUR ZONE MIXING MODELS	5.21- Re-circulation Case	5.22- Partitioned Case
	 	
a. Contaminant injected in 1  	$< C(\infty) > = \frac{q_1}{Q}$ $C_e(\infty) = \frac{q_1}{Q}$ $\varepsilon^c = 1$ $\eta^c = 0.5$	$< C(\infty) > = \frac{q_1}{Q}$ $C_e(\infty) = \frac{q_1}{Q}$ $\varepsilon^c = 1$ $\eta^c = 0.5$
b. Contaminant injected in 2  	$< C(\infty) > = \frac{q_2}{Q}\cdot\left(\frac{3+4\beta}{4+4\beta}\right)$ $C_e(\infty) = \frac{q_2}{Q}$ $\varepsilon^c = \frac{4+4\beta}{3+4\beta}$ $\eta^c = \frac{4+4\beta}{7+8\beta}$	$< C(\infty) > = \frac{q_2}{Q}\cdot\left(\frac{4\beta^3+6\beta^2+4\beta+1}{4\beta(1+\beta)^2}\right)$ $\varepsilon^c = \frac{4\beta(1+\beta)^2}{4\beta^3+6\beta^2+4\beta+1}$ $\eta^c = \frac{4\beta(1+\beta)^2}{8\beta^3+14\beta^2+8\beta+1}$
c. Contaminant injected in 3  	$< C(\infty) > = \frac{q_3}{Q}\cdot\left(\frac{1+2\beta}{2+2\beta}\right)$ $C_e(\infty) = \frac{q_3}{Q}$ $\varepsilon^c = \frac{2+2\beta}{1+2\beta}$ $\eta^c = \frac{2+2\beta}{3+4\beta}$	$< C(\infty) > = \frac{q_3}{Q}\cdot\left(\frac{4\beta^2+5\beta+2}{4(1+\beta)^2}\right)$ $\varepsilon^c = \frac{4(1+\beta)^2}{4\beta^2+5\beta+2}$ $\eta^c = \frac{4(1+\beta)^2}{8\beta^2+13\beta+6}$
d. Contaminant injected in 4  	$< C(\infty) > = \frac{q_4}{Q}\cdot\left(\frac{1+4\beta}{4\beta}\right)$ $C_e(\infty) = \frac{q_4}{Q}$ $\varepsilon^c = \frac{4\beta}{1+4\beta}$ $\eta^c = \frac{4\beta}{1+8\beta}$	$< C(\infty) > = \frac{q_4}{Q}\cdot\left(\frac{3+4\beta}{4+4\beta}\right)$ $C_e(\infty) = \frac{q_4}{Q}$ $\varepsilon^c = \frac{4+4\beta}{3+4\beta}$ $\eta^c = \frac{4+4\beta}{7+8\beta}$

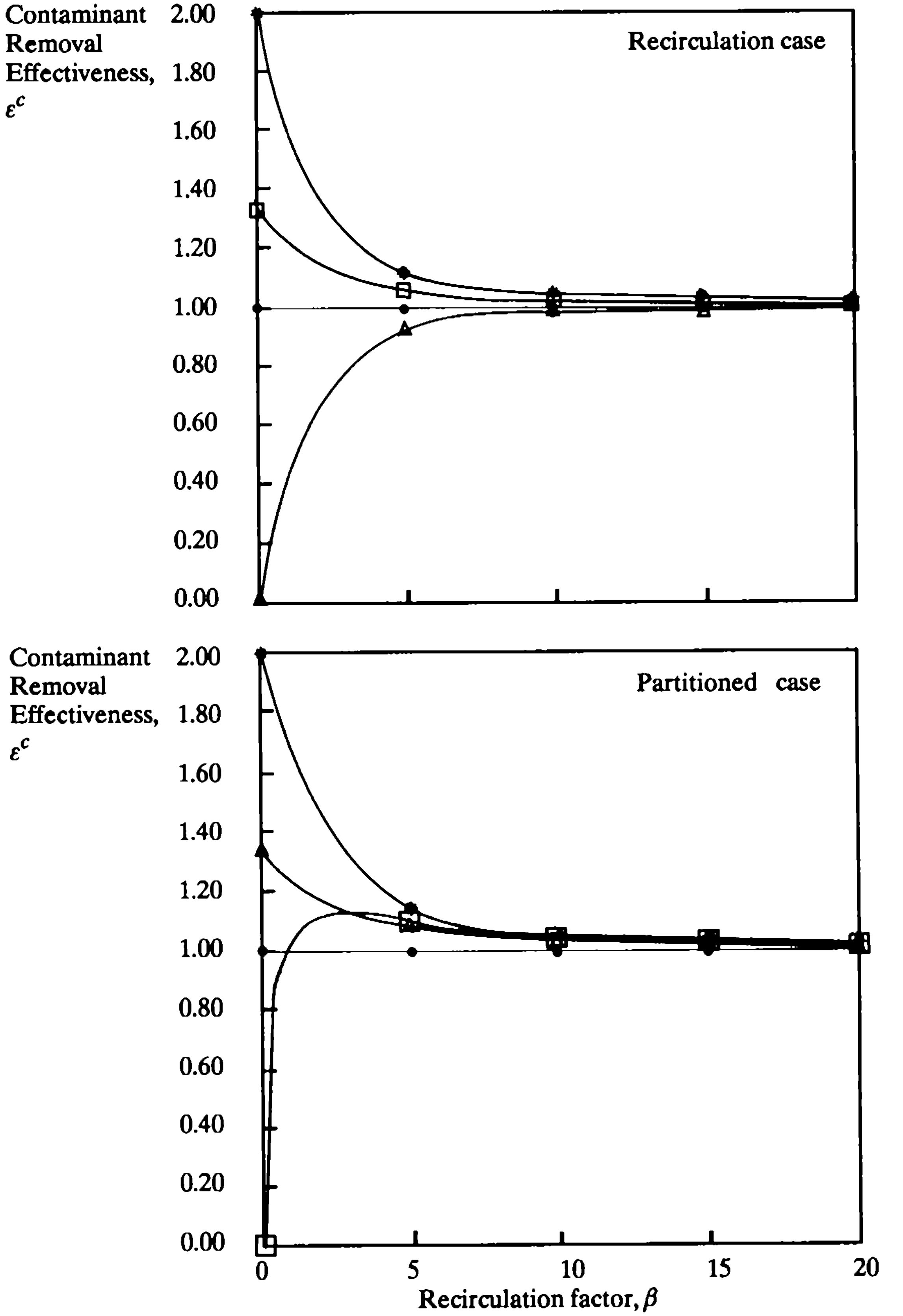


Figure 5.2 - Contaminant removal effectiveness vs recirculation factor.

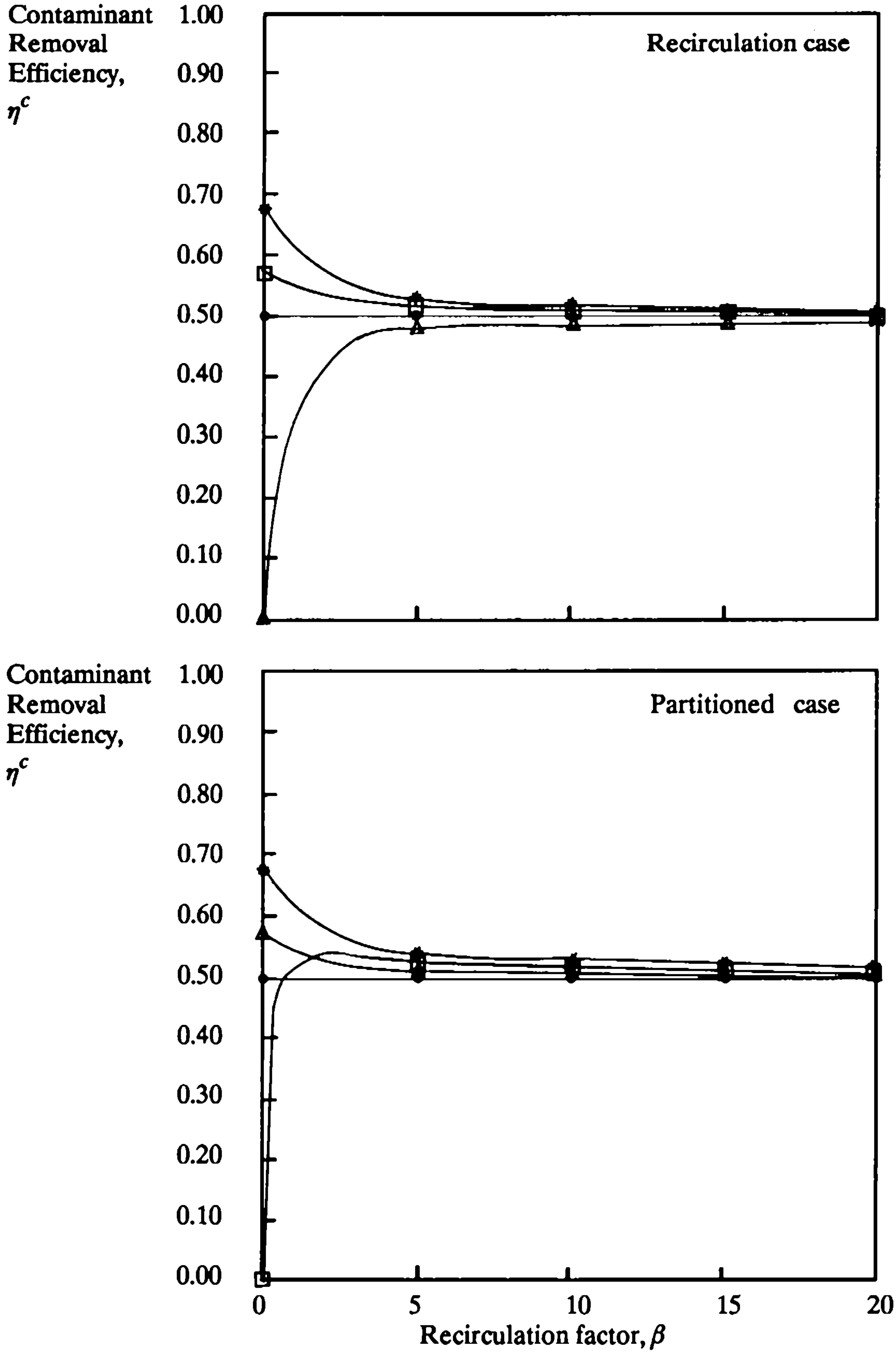


Figure 5.3 - Contaminant Removal Efficiency vs recirculation factor.

Key:	------●------	------□------	------◆------	------▲------
	case a	case b	case c	case d

6. THE USE OF FLUID FLOW MODELS FOR DETERMINING THE EFFECTIVENESS OF A VENTILATION SYSTEM.

Sections 3, 4 and 5 illustrate the derivation of contaminant removal effectiveness indices for a room which is being modelled in increasing detail, from 1 to 2 and then to 4 zones. In principle, this process could be continued to any number of desired zones. However, a more detailed analysis of the air flow patterns within a room can be better obtained by solving, in 3 dimensions, the relevant fluid flow equations. The solution can provide air velocities and contaminant concentrations throughout the space, from which it is possible to derive the indices associated with air change efficiency and contaminant removal effectiveness. Unfortunately, such solutions can only be obtained by means of complex computer programs which usually require substantial computing resources for their implementation.

Liddament [10] has reviewed the principles of this method of air flow simulation, and also provides a list of available computer codes.

7. FREQUENCY DISTRIBUTION AND MEAN AGE

7.1 Frequency Distribution

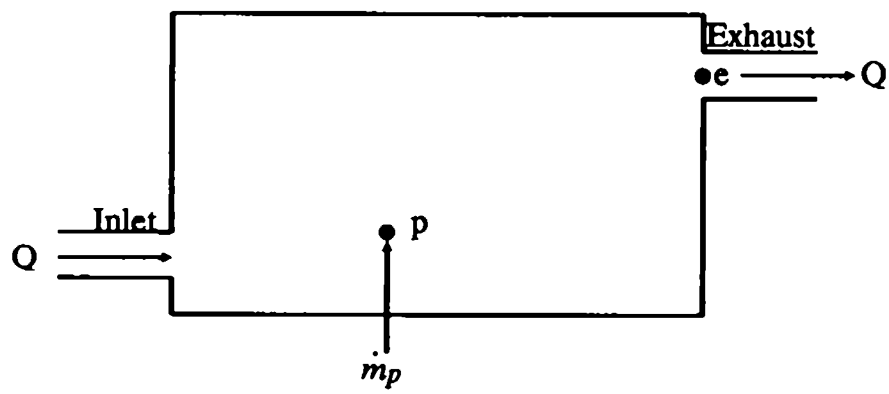


Fig. 7.1 Representation of a mechanically ventilated room.

Figure (7.1) represents a mechanically ventilated room with one inlet duct and one exhaust duct. Let a contaminant be injected continuously in this room at a rate $\dot{m}_p$, starting from time $\tau = 0$. At any point p, the contaminant will have an internal age distribution $A_p(t)$, as shown in figure 7.2, where t is the age of the contaminant at some moment in time τ. If p is taken to be at the exhaust duct, $A_p(t)$ may be called the exit age distribution $A_e(t)$. Note that, since $A_p(t)$ is a frequency distribution, the $\int_0^\infty A_p(t)\,.\,dt = 1$.

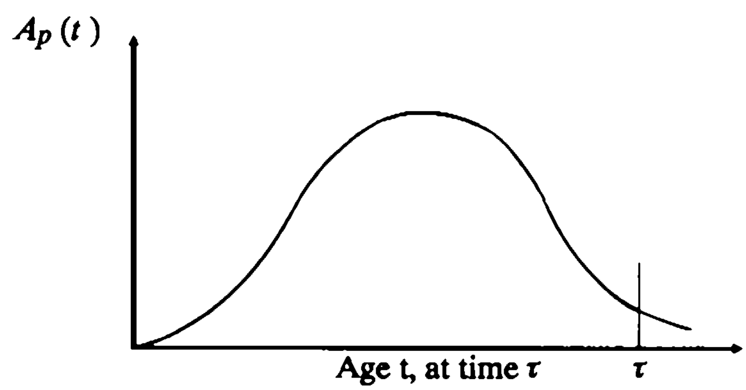


Fig. 7.2 Frequency distribution curve for the contaminant in the room.

7.2 Definition of the Mean Ages

The local mean age of contaminant is defined as the average time it takes for the contaminant to travel from its source of emission to any point p in the room. The mean age of the contaminant at the point p can be found from the centroid of the frequency distribution curve, by taking moments about the vertical axis.

The mean age of the contaminant at p, $\bar{\tau}_p^c = \dfrac{\int_0^\infty t \,.\, A_p(t) \,.\, dt}{\int_0^\infty A_p(t) \,.\, dt}$ (7.1)

As $\int_0^\infty A_p(t) \,.\, dt = 1.$, $\bar{\tau}_p^c$ is thus:

$$\bar{\tau}_p^c = \int_0^\infty t \,.\, A_p(t) \,.\, dt \tag{7.2}$$

The local mean age of contaminant will be different for different points in the room. For example, the mean age of the contaminant at the exhaust duct can be expressed by:

$$\bar{\tau}_e^c = \int_0^\infty t \,.\, A_e(t) \,.\, dt \tag{7.3}$$

The room mean age of contaminant, $<\bar{\tau}^c>$ is the average value of the local mean ages of contaminant for all points in a room.

7.3 Relationship between τ_n^c and $\bar{\tau}_e^c$

The distribution curve may be used to derive a useful relationship between the nominal time constant for the contaminant and the local mean age of the contaminant at the exhaust duct. Let A(t) be the average internal age distribution at time τ for the whole room.

The total amount of contaminant in the room at any time τ is:

$$m \,.\, \int_0^\infty A(t) \,.\, dt \tag{7.4}$$

The total amount leaving the room at some moment over an interval of time δt is:

$$M = \dot{m} \,.\, \int_0^\infty A_e(t) \,.\, dt \tag{7.5}$$

Consider a period of time $\tau = 0$ to $\tau = T$.

The amount of contaminant injected is $\dot{m} \,.\, T$.

The amount of contaminant leaving is $\int_0^T M \,.d\tau = \dot{m} \,.\int_0^T \int_0^\infty A_e(t) \,.dt \,.d\tau$

Hence, the contaminant balance equation is:

$$\dot{m} \,.\, T = m \int_0^\infty A(t)\,.dt + \dot{m} \int_0^T \int_0^\infty A_e(t)\,.dt\,.d\tau \tag{7.6}$$

If the injection starts at time $\tau = 0$, then at time $\tau = \mathrm{T}$, equation 7.6 may be written:

$$\dot{m} = m \int_0^T A(t)\,.dt + \dot{m} \int_0^T \int_0^T A_e(t)\,.dt\,.dt \tag{7.7}$$

Differentiating and rearranging equation 7.7 gives:

$$A_e(t) = -\frac{m}{\dot{m}} \,.\, A'(t) \tag{7.8}$$

Substituting equation 2.3 into equation 7.6 gives:

$$A_e(t) = -\tau_n^c \,.\, A'(t) \tag{7.9}$$

The mean exit age of the contaminant, $\bar{\tau}_e^c$ can then be found:

$$\bar{\tau}_e^c = \int_0^\infty A_e(t)\,.t\,.dt \tag{7.10}$$

Substituting equation 7.9 back into equation 7.10 gives:

$$\bar{\tau}_e^c = -\int_0^\infty \tau_n^c \,.\, A'(t)\,.t\,.dt$$

Which in turn may be written:

$$\bar{\tau}_e^c = -\tau_n^c \,.\int_0^\infty A'(t)\,.t\,.dt$$

Where: $\int_0^\infty A'(t)\,.t\,.dt = [A(t)\,.t]_0^\infty - \int_0^\infty A(t)\,.dt = -1$

Hence:

$$\bar{\tau}_e^c = \tau_n^c \tag{7.11}$$

8. METHODS OF MEASUREMENT

In order to measure any of the indices, it is necessary to measure the concentration of the contaminant at the appropriate points in the room. If it is not possible to measure the contaminant itself, then a tracer gas which imitates the behaviour of the contaminant may be used instead.

The indices may be obtained either by measuring the equilibrium concentration due to continuous injection of contaminant at a constant rate, or by monitoring contaminant history due to different methods of contaminant injection.

8.1 Measurement of Equilibrium Contaminant Concentration

All the indices described in section 2 may be obtained by measurement of the equilibrium concentration due to continuous injection of contaminant. Although it is simple and direct, this method has some disadvantages, such as

(i) The time taken to reach equilibrium may be long, requiring the injection of a large quantity of contaminant,

(ii) It is often difficult to measure the room average contaminant concentration, $< C(\infty) >$, because it is difficult to identify sufficient points of measurement from which a representative average can be obtained,

(iii) It is difficult to measure the concentration of the contaminant close to its point of injection, as required for the direct determination of U_p and D_p.

8.2 Measurement of Contaminant History

The indices which can be obtained from a measurement of contaminant history are:

(i) the Total Dosage Index,

(ii) the Transfer Index,

(iii) the Local Mean Age of the contaminant at any point in the room (including the exhaust duct),

(iv) the Room Mean Age of the contaminant.

The equations for Dosage Index and Transfer Index have already been given in section 2.

The measurement of Local and Room Mean Age of contaminant follows the same theory and practice as for air change efficiency. The only difference is that in this case, contaminant (or an equivalent tracer gas) is injected at the appropriate point in the room, rather than in the inlet duct.

The tracer injection may be by Pulse, Step-Up, or Decay. The same equations apply as for air change efficiency. These are given in AIVC TN-28 [9], and are:

Local Mean Age of Contaminant ($\bar{\tau}_p^c$)

1. Pulse method $$\bar{\tau}_p^c = \frac{\int_0^\infty t \,.\, C_p(t) \,.\, dt}{\int_0^\infty C_p(t) \,.\, dt}$$

2. Step-Up method $$\bar{\tau}_p^c = \int_0^\infty \left(1 - \frac{C_p(t)}{C_s}\right) .\, dt$$

3. Decay method $$\bar{\tau}_p^c = \int_0^\infty \frac{C_p(t)}{C(0)} \,.\, dt$$

Room Mean Age of Contaminant $< \bar{\tau}^c >$

1. Pulse method $$< \bar{\tau}^c > = \frac{Q}{2V} \,.\, \left(\frac{\int_0^\infty t^2 \,.\, C_e(t) \,.\, dt}{\int_0^\infty C_e(t) \,.\, dt}\right)$$

2. Step-Up method $$< \bar{\tau}^c > = \frac{Q}{V} \,.\, \int_0^\infty t \,.\, \left(1 - \frac{C_e(t)}{C_s}\right) .\, dt$$

3. Decay method $$< \bar{\tau}^c > = \frac{Q}{V} \,.\, \int_0^\infty t \,.\, \frac{C_e(t)}{C(0)} \,.\, dt$$

$$< \bar{\tau}^c > = \frac{\int_0^\infty t \,.\, C_e(t) \,.\, dt}{\int_0^\infty C_e(t) \,.\, dt}$$

There is a difference in the application of the decay method. With air change efficiency, it is necessary to begin the decay from a uniform concentration throughout the room. However, in this case, the decay begins from the equilibrium distribution of contaminant concentration, which in general is not uniform.

When the local mean age of the contaminant is measured at the exhaust duct, the result also gives, by equation 7.11, the nominal time constant for the contaminant, that is:

$$\bar{\tau}_p^c = \bar{\tau}_e^c = \tau_n^c$$

The quantities obtained from measurements of contaminant history may also be used to determine the contaminant removal effectiveness of the room, ε^c , and the local air quality index, ε_p^c . This avoids the difficulty of measuring the equilibrium room average contaminant concentration, $< C(\infty) >$. In the case of contaminant removal effectiveness it is also necessary to know the nominal time constant for the ventilation air, τ_n ; ε^c may then be found from equation 2.7. The local air quality index, ε_p^c , may be found from either the total dosage index, D_p , or the transfer index, T_{pn} , using the appropriate equations from section 2.8. The local purging flow rate, U_p , may in turn be found from ε_p^c using an equation from section 2.8.

9. INTERPRETATION AND APPLICATIONS

The contaminant removal effectiveness indices fall into two groups. The first, which comprises the contaminant removal effectiveness, ε^c, the contaminant removal efficiency, η^c, and the nominal time constant of the contaminant, τ_n^c, are all indices of a whole room and its ventilation system. The second, which comprises the local air quality index, ε_p^c, the local purging flow rate, U_p, the dosage indices, D_p and D_{pn}, and the transfer index, T_{pn}, are all indices which refer to conditions at a specific point or small volume within a room.

9.1 Room Indices

The indices ε^c, η^c and τ_n^c are connected to each other by the equations given in section 2. It is sufficient therefore, to use only one of them. The nominal time constant of the contaminant is the least valuable as it provides little information unless the nominal time constant of the ventilation air is also known. The contaminant removal efficiency has the advantage that its value is contained within the range 0 to 1, but there is no simple association between its value and the quantity of contaminant in a room. The contaminant removal effectiveness on the other hand can easily be interpreted in terms of contaminant ratios, and is the preferred index here.

From the examples given in sections 3, 4 and 5, it is clear that the minimum acceptable value of the contaminant removal effectiveness for any room and its ventilation system should normally be $\varepsilon^c = 1$. A lower value shows that the ventilation system is not as good as the simple fully mixed case in ventilating the whole room. Where the contaminant source is uniformly distributed, as would often be the case if the occupants themselves were the main source, the maximum possible contaminant removal effectiveness is achieved in pure piston flow, with a value of $\varepsilon^c = 2$. Thus, for many applications, it may be expected that ε^c should be within the range 1 to 2.

When the contaminant source is not uniformly distributed, the examples given in sections 3.22, 3.23 and 3.24 can be used as a guide to the maximum attainable value of ε^c. However, as was pointed out in section 3.3, interpretation of a particular value of ε^c must take into account the contaminant injection pattern and the position of the occupants. A high value for ε^c is not sufficient to show that the ventilation system is satisfactory.

The prediction of contaminant removal effectiveness values for real ventilation systems may often be possible by direct comparison with one of the idealised examples of sections 3, 4 and 5. If this is not possible, a multi-zone model may be set up and analysed in the manner of section 5, where the zones may be subdivisions of a single space, or complete rooms. Where several zones are served by a central air handling unit with recirculation, the recirculation loop can be treated as an extra zone. For example, the short-circuiting flow case described in section 4.21(a) would be equivalent to a single zone with recirculation if zone 1 was taken to be the occupied zone and zone 2 was the recirculation loop.

The results of section 3.2 for pure piston flow may also be used to aid the interpretation of both measured and predicted contaminant removal effectiveness values. The value for piston flow which is closest to the real case in terms of contaminant injection distribution may be obtained from figure 3.10, and this is the maximum attainable value. The ratio, r^c, of the real value to this maximum value is an indication of how close the ventilation system is to achieving optimum contaminant removal.

9.2 Local Indices

The local air quality index and the local purging flow rate are closely related. However, according to its formal definition, the local purging flow rate applies only when contaminant is released within the elementary volume surrounding the point of reference. Because of this, and also because ε_p^c is the local equivalent of ε^c, it is recommended that the local air quality index be used to describe conditions at a point.

As with contaminant removal effectiveness, the minimum acceptable value of the local air quality index should be taken as $\varepsilon_p^c = 1$. Values less than this indicate poor contaminant removal from the reference point, and any value greater than this shows that conditions at the reference point are better than they would be in fully mixed flow.

Prediction of local air quality index values may also be made by direct comparison with the examples of sections 3, 4 and 5, or by setting up and analysing a multi-zone model, and the results of section 3.2 for pure piston flow may again be used to aid interpretation.

The contaminant removal effectiveness and the local air quality index provide valuable information on the performance of a ventilation system. They do not provide any information on the level of exposure of an occupant to contaminants. The exposure level can be measured by the dosage indices, D_p and D_{pn}. Maximum values of D_p and D_{pn} must be obtained for specific contaminants from the recommendations of appropriate bodies.

The transfer index is essentially the dosage at a point p due to the release of unit quantity of contaminant at a point n. It therefore fulfills a similar function to D_{pn}.

9.3 Parametric Models

The purpose of the analyses carried out in sections 3, 4 and 5 was to express contaminant removal effectiveness indices in terms of the model parameters. A complex model with a large number of parameters will give a better representation of the real situation, but will be less convenient to use. There is an advantage in finding models with a minimum number of parameters which will nevertheless provide acceptable modelling of most ventilation systems.

Many ventilation systems create a pattern of air movement which consists of a piston-like flow of air from a set of inlet ducts to a set of outlet ducts, with internal mixing superimposed upon it. The two zone mixing model described in section 4 is one way of modelling this pattern of air movement. The model is essentially a single parameter model, as its behaviour is mostly determined by the value of the recirculation factor.

However, the premise that a room can be adequately modelled by two geometrically distinct zones may not always be valid. An alternative model can be constructed by assuming that the air movement pattern is a combination of piston flow and fully mixed flow in equal proportions, their relative importance being given by their time constants. The contaminant removal effectiveness may then be expressed in terms of time constants. There are four nominal time constants which may be defined as follows:

τ_{nm} for fully mixed ventilation air,
τ^c_{nm} for the contaminant in fully mixed flow,
τ_{np} for piston flow ventilation air, and
τ^c_{np} for the contaminant in piston flow.

The average time constants may be defined as:

$$< \tau_n > = \frac{1}{2} . \tau_{np} + \frac{1}{2} . \tau_{nm}$$

$$< \tau^c_n > = \frac{1}{2} . \tau^c_{np} + \frac{1}{2} . \tau^c_{nm}$$

The latter simplifies, because in fully mixed flow, $\tau_{nm} = \tau^c_{nm}$, and so the contaminant removal effectiveness may be written:

$$\varepsilon^c = \frac{< \tau_n >}{< \tau^c_n >} = \frac{\tau_{np} + \tau_{nm}}{\tau^c_{np} + \tau_{nm}}$$

As it is the relative magnitudes of the time constants that is important, we may further simplify by writing:

$$S = \frac{\tau_{np}}{\tau_{nm}} \quad \text{and} \quad S^c = \frac{\tau^c_{np}}{\tau_{nm}}$$

and so:

$$\varepsilon^c = \frac{S + 1}{S^c + 1}$$

This is a two parameter model. It is not a zonal model, and therefore it avoids the need to split the room into distinct zones. It provides better modelling than the two zone model when there is a strong element of piston (or displacement) flow in the air movement pattern, and it has the advantage that the parameters which describe its behaviour, S and S^c, can be easily understood.

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APPENDIX A

The nominal time constant for the contaminant is represented by the symbol τ_n^c in most papers given as references. It was decided to use the same symbol as for the nominal time constant for the ventilation air, adding a superscript c which shows that the index refers to the contaminant.

The contaminant removal effectiveness is represented by the symbol $< \varepsilon_v^c >$ in most papers given as references. In this report, the symbol was simplified to ε^c, where the superscript c shows that the index refers to the contaminant.

The contaminant removal efficiency is represented by the symbol $< \eta_v^c >$ in most papers given as references. In this report, the symbol was simplified to η^c, where the superscript c shows that the index refers to the contaminant.

The local air quality index is represented by the symbol ε_v in most papers given as references. In this report, the symbol was modified to ε_p^c. The superscript c shows that the index refers to the contaminant, and the subscript p shows that the index refers to a point p in the room.

The transfer index is represented by the symbol T_i in most papers given as references. In this report, the index was modified to T_{pn}, where the subscript pn shows that the index refers to a transfer of contaminant between two points in the room, namely points n and p.

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Air Infiltration and Ventilation Centre

University of Warwick Science Park,
Barclays Venture Centre,
Sir William Lyons Road,
Coventry CV4 7EZ,
Great Britain.

Telephone: +44 (0) 203 692050
Fax: +44 (0) 203 416306

Operating Agent for International Energy Agency, the Oscar Faber Partnership, Upper Marlborough Road, St. Albans, UK